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Quantum Inconsistency of Einstein Supergravity

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Abstract

We show that $N = 1$, $D = 4$ Einstein-frame supergravity is inconsistent at one loop because of an anomaly in local supersymmetry transformations. A Jacobian must be added to the Einstein-frame Lagrangian to cancel this anomaly. We show how the Jacobian arises from the super-Weyl field redefinition that takes the superspace Lagrangian to the Einstein frame. We present an explicit example which demonstrates that the Jacobian is necessary for one-loop scattering amplitudes to be frame independent.

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1 Introduction

The component Lagrangian of matter-coupled supergravity can be derived from a superspace formulation or a tensor calculus [1, 2]. Both approaches inevitably lead to a component theory in which the gravitational action is of a generalized Brans-Dicke form,

$$e^{-1}\mathcal{L} = -\frac{1}{2}e^{-K/3}\mathcal{R} + \dots \quad (1.1)$$

In this expression, e is the vielbein determinant, \mathcal{R} is the curvature scalar and K is the Kähler potential, a function of the scalar fields A^*, A .

The Lagrangian (1.1) leads to a kinetic mixing between the graviton and the scalar fields. In addition, the kinetic terms of the scalar fields appear in non-Kähler form. It is therefore convenient and customary to carry out a field-dependent Weyl rescaling of the metric to bring the Lagrangian into canonical Einstein form. In supergravity, this Weyl rescaling must also be accompanied by a chiral rotation of the fermions. As we shall see, this rotation gives rise to an anomalous Jacobian in the supergravity action.

The field redefinitions needed to go to this “Einstein frame” are usually performed in terms of component fields [1, 2]. This obscures the symmetries of the theory and complicates the study of anomalies and their consequences. Therefore, in this paper we will use the superspace approach of [2] to study anomalies in supergravity theories. We will show that:

1. The Einstein-frame field redefinitions can be carried out directly in superspace through a super-Weyl transformation of the vielbein. The corresponding component-field Lagrangian gives rise to ordinary Einstein gravity.
2. Local supersymmetry transformations in the Einstein frame involve chiral rotations of the fermions. They are anomalous at one loop.
3. The local supersymmetry anomaly is cancelled by a Jacobian that arises from the transition to the Einstein frame. This Jacobian is necessary to ensure the quantum consistency of Einstein-frame supergravity.
4. The anomalous Jacobian can have important physical consequences. For example, it is necessary to ensure the quantum equivalence of scattering amplitudes computed in different frames.

This paper is organized as follows. In Section 2 we define super-Weyl transformations and derive the corresponding Jacobians. In Section 3 we study the transition to the Einstein frame. We show that one-loop supergravity invariance of the Einstein-frame Lagrangian requires that a certain superspace Jacobian be added to the bare Lagrangian. In Section 4 we present an explicit example which illustrates the physical importance of this Jacobian. We summarize our results in Section 5.

2 Super-Weyl Transformations

2.1 Classical Level

In this section we study super-Weyl transformations in classical and quantum supergravity. These transformations will play an important role throughout this paper.

In what follows we use the notation and conventions of [2]. We take the matter-coupled supergravity Lagrangian to be of the form

$$\begin{aligned} \mathcal{L}(X) = & \int d^2\Theta \, 2\mathcal{E} \left[\frac{3}{8} (\bar{\mathcal{D}}^2 - 8R) \exp \left\{ -\frac{1}{3} (K(\Phi^\dagger, \Phi) + \Gamma(\Phi^\dagger, \Phi, V)) \right\} \right. \\ & \left. + \frac{1}{4} H_{ab}(\Phi) W^{(a)} W^{(b)} + P(\Phi) \right] + \text{h.c.} , \end{aligned} \quad (2.1)$$

where $X = (\Phi, V, E_M^A)$ denotes the set of fields in the supergravity Lagrangian, Φ and V are the chiral and vector superfields, and E_M^A is the supervielbein. In this expression, K is the Kähler potential, P the superpotential, and $W^{(a)}$ the field strength superfield, where (a) is the index for the adjoint representation.^{#1} In addition, H_{ab} is the gauge kinetic function, and Γ is the gauge counterterm which renders the Lagrangian gauge invariant.

In the superspace formalism, supergravity transformations are given by translations in superspace. Chiral and vector superfields transform as follows,

$$\delta_{\text{SUSY}} \Phi = -\xi^A \mathcal{D}_A \Phi , \quad \delta_{\text{SUSY}} V = -\xi^A \mathcal{D}_A V , \quad (2.2)$$

while the vielbein transforms as

$$\delta_{\text{SUSY}} E_M^A = -\mathcal{D}_M \xi^A - \xi^B T_{BM}^A . \quad (2.3)$$

In these expressions, the \mathcal{D}_A are covariant derivatives and the T_{BM}^A are the superspace torsion. The superspace formalism ensures that the Lagrangian (2.1) is invariant,

$$\mathcal{L}(X + \delta_{\text{SUSY}} X) = \mathcal{L}(X) , \quad (2.4)$$

up to a total derivative, under the supersymmetry transformations (2.2) and (2.3).

Super-Weyl transformations are defined as rescalings of the superspace vielbein that preserve the torsion constraints [3].^{#2} They are parameterized by a chiral superfield Σ . If we denote the super-Weyl transformed X field as \hat{X} , we can write

$$X = \hat{X} + \delta_{\text{SW}} X , \quad (2.5)$$

^{#1}In this paper, we sometimes omit the spin index α . Therefore $W^{(a)} W^{(b)}$ should be understood as $W^{(a)\alpha} W_\alpha^{(b)}$.

^{#2}It is important to distinguish between super-Weyl and super-Weyl-Kähler transformations. The latter are symmetries of the classical supergravity Lagrangian. A super-Weyl-Kähler transformation is a super-Weyl transformation, with chiral superfield parameter Σ , combined with a redefinition of the Kähler potential and superpotential, $K \rightarrow K + 6\Sigma + 6\Sigma^\dagger$, $P \rightarrow \exp(-6\Sigma)P$.

where $\delta_{\text{SW}}X$ is the super-Weyl variation of X .

To linear order in Σ , the super-Weyl transformations are given by

$$\begin{aligned}\delta_{\text{SW}}\mathcal{E} &= 6\Sigma\mathcal{E} + \frac{\partial}{\partial\Theta^\alpha}(S^\alpha\mathcal{E}) \\ \delta_{\text{SW}}\Phi &= -S^\alpha\frac{\partial}{\partial\Theta^\alpha}\Phi \\ \delta_{\text{SW}}(\bar{\mathcal{D}}^2 - 8R)U &= -(\bar{\mathcal{D}}^2 - 8R)(4\Sigma - 2\Sigma^\dagger)U - S^\alpha\frac{\partial}{\partial\Theta^\alpha}(\bar{\mathcal{D}}^2 - 8R)U \\ \delta_{\text{SW}}W_\alpha &= -3\Sigma W_\alpha - S^\beta\frac{\partial}{\partial\Theta^\beta}W_\alpha,\end{aligned}\tag{2.6}$$

where U is any real vector superfield of Weyl weight zero, and S^α is defined by

$$S^\alpha = \Theta^\alpha(2\Sigma^\dagger - \Sigma)| + \Theta^2(\mathcal{D}^\alpha\Sigma)|.\tag{2.7}$$

The bar $|$ denotes the $\theta = \bar{\theta} = 0$ component of the superfield.

Note that super-Weyl transformations also induce chiral rotations of the component fermions. Taking the appropriate components of Eq. (2.6) and re-exponentiating, we find

$$\chi = \exp(\Sigma| - 2\Sigma^\dagger|)\hat{\chi}, \quad \lambda = \exp(-3\Sigma|)\hat{\lambda},\tag{2.8}$$

where χ and λ are the fermions in the chiral and gaugino multiplets, respectively.

In general, super-Weyl transformations are not symmetries of the classical supergravity Lagrangian. Indeed, substituting the transformed variables (2.6) into the Lagrangian (2.1), all derivative terms cancel. Nevertheless, a nontrivial Σ dependence remains. At the classical level, the Lagrangian (2.1) becomes

$$\begin{aligned}\mathcal{L}(\hat{X} + \delta_{\text{SW}}X) &= \int d^2\Theta\, 2\hat{\mathcal{E}}\left[\frac{3}{8}\left(\hat{\mathcal{D}}^2 - 8\hat{R}\right)\exp\left\{-\frac{1}{3}\left(\hat{K} - 6\Sigma - 6\Sigma^\dagger + \hat{\Gamma}\right)\right\}\right. \\ &\quad \left.+ \frac{1}{4}\hat{H}_{ab}\hat{W}^{(a)}\hat{W}^{(b)} + \exp(6\Sigma)\hat{P}\right] + \text{h.c.},\end{aligned}\tag{2.9}$$

where the hatted objects are evaluated using the Weyl-transformed fields. The Kähler and superpotential are different, so the Lagrangian (2.1) is not invariant.

2.2 Quantum Level

We are now ready to discuss the anomalous Jacobian associated with a given super-Weyl transformation. A proper framework is provided by the one-particle-irreducible (1PI) effective Lagrangian \mathcal{L}_{1PI} , defined by

$$\int d^4x\, \mathcal{L}_{\text{1PI}}(X_{\text{C}}) = -i\log\left[\int[dX_{\text{Q}}]\exp\left(i\int d^4x\, \mathcal{L}_{\text{bare}}(X_{\text{C}} + X_{\text{Q}})\right)\right].\tag{2.10}$$

Here $\mathcal{L}_{\text{bare}}$ is the bare Lagrangian, and X_C and X_Q are classical and quantum parts of the X field, respectively.

In general, super-Weyl transformations are anomalous; they have a mixed super-Weyl-gauge anomaly.^{#3#4} Anomalies generate a set of non-local terms in the 1PI effective action [5]. For the case at hand, the one-loop anomaly-induced terms are [6, 7],

$$\Delta\mathcal{L} = -\frac{1}{256\pi^2} \int d^2\Theta \, 2\mathcal{E} \, W^{(a)}W^{(a)} \, \frac{1}{\square} (\bar{\mathcal{D}}^2 - 8R) \left[4(T_R - 3T_G)R^\dagger - \frac{1}{3}T_R\mathcal{D}^2K \right] + \text{h.c.} , \quad (2.11)$$

where we have omitted a term from the sigma-model anomaly that is irrelevant for our discussion. In Eq. (2.11), T_G is the Dynkin index of the adjoint representation, normalized to N for $SU(N)$, and T_R is the Dynkin index associated with the matter fields. A sum over all matter representations is understood. The first term, which contains the R^\dagger superfield, arises from the superconformal anomaly. It is proportional to the beta function coefficient, $b_0 = 3T_G - T_R$. The second term expresses the Kähler anomaly [6, 7].

The variation of $\Delta\mathcal{L}$ can be computed by considering a super-Weyl transformation with superfield parameter Σ . Under such a transformation, the superfield R changes as follows,

$$\delta_{\text{SW}}R = -2(2\Sigma - \Sigma^\dagger)R - \frac{1}{4}\bar{\mathcal{D}}^2\Sigma^\dagger . \quad (2.12)$$

This induces a shift by Σ in the R^\dagger term in Eq. (2.11). A second shift comes from replacing K by $\hat{K} - 6\Sigma - 6\Sigma^\dagger$. These two shifts induce the following change in $\Delta\mathcal{L}$,

$$\Delta\mathcal{L} \rightarrow \Delta\mathcal{L} + \mathcal{L}_J , \quad (2.13)$$

where

$$\mathcal{L}_J = \frac{1}{16\pi^2} (3T_R - 3T_G) \int d^2\Theta \, 2\mathcal{E} \, \Sigma W^{(a)}W^{(a)} + \text{h.c.} \quad (2.14)$$

The Lagrangian \mathcal{L}_J can be interpreted as the superspace Jacobian that arises from the super-Weyl transformation (2.5). (Note that the imaginary part of $\Sigma|$ corresponds to the Jacobian from the anomalous $U(1)_R$ transformation.)

The nonvanishing Jacobian implies that the functional measure is not invariant. It transforms as follows under an arbitrary super-Weyl transformation:

$$[d\Phi][dV] = [d(\hat{\Phi} + \delta_{\text{SW}}\Phi)][d(\hat{V} + \delta_{\text{SW}}V)] = [d\hat{\Phi}][d\hat{V}] \exp \left[i \int d^4x \, \mathcal{L}_J \right] , \quad (2.15)$$

The super-Weyl-rescaled 1PI Lagrangian is then

$$\exp \left[i \int d^4x \, \mathcal{L}_{\text{1PI}} \right] = \int [d(\hat{X} + \delta_{\text{SW}}X)] \exp \left[i \int d^4x \, \mathcal{L}_{\text{bare}}(\hat{X} + \delta_{\text{SW}}X) \right]$$

^{#3}It also has a mixed super-Weyl-gravity anomaly. We ignore the gravity anomaly here.

^{#4}For a discussion of supergravity anomalies in the compensator formalism, see [4].

$$\begin{aligned}
&= \int [d\hat{X}] \exp \left[i \int d^4x \left\{ \mathcal{L}_{\text{bare}}(\hat{X})|_{\hat{K} \rightarrow \hat{K} - 6\Sigma - 6\Sigma^\dagger, \hat{P} \rightarrow e^{6\Sigma} \hat{P}} + \mathcal{L}_J \right\} \right] \\
&= \int [d\hat{X}] \exp \left[i \int d^4x \hat{\mathcal{L}}_{\text{bare}}(\hat{X}) \right] , \tag{2.16}
\end{aligned}$$

where

$$\hat{\mathcal{L}}_{\text{bare}}(\hat{X}) \equiv \mathcal{L}_{\text{bare}}(\hat{X} + \delta_{\text{SW}} X) + \mathcal{L}_J = \mathcal{L}_{\text{bare}}(\hat{X})|_{\hat{K} \rightarrow \hat{K} - 6\Sigma - 6\Sigma^\dagger, \hat{P} \rightarrow e^{6\Sigma} \hat{P}} + \mathcal{L}_J . \tag{2.17}$$

In these expressions, $\hat{\mathcal{L}}_{\text{bare}}$ is the bare Lagrangian for the quantum theory with super-Weyl-rescaled variable \hat{X} . The bare Lagrangian does not contain the anomaly term $\Delta\mathcal{L}$, which arises from integrating out the massless quantum fields. It does, however, contain the Jacobian \mathcal{L}_J . As we will see, \mathcal{L}_J is important for ensuring the quantum consistency of supergravity in the Einstein frame.

3 Einstein Supergravity

3.1 The Einstein Frame

In this section, we find the field-dependent Weyl rescaling that takes the “supergravity frame” Lagrangian of Eq. (2.1)

$$e^{-1}\mathcal{L} = -\frac{1}{2}e^{-K/3}\mathcal{R} + \dots \tag{3.1}$$

into the Einstein frame. In the literature, this rescaling has traditionally been done in terms of component fields [1, 2]. Here we perform the transformation in superspace. This allows us to keep better track of the symmetries of the theory.

The relevant superfield rescaling is, as we will see below, a super-Weyl transformation with transformation parameter Σ_E ,

$$\begin{aligned}
\mathcal{L}_E(X) &= \int d^2\Theta \, 2\mathcal{E} \left[\frac{3}{8}(\bar{\mathcal{D}}^2 - 8R) \exp \left\{ -\frac{1}{3}(K - 6\Sigma_E - 6\Sigma_E^\dagger + \Gamma) \right\} \right. \\
&\quad \left. + \frac{1}{4}H_{ab}W^{(a)}W^{(b)} + \exp(6\Sigma_E)P \right] + \text{h.c.} , \tag{3.2}
\end{aligned}$$

where we have omitted all “hats” in the above equation. (Here and hereafter, all quantities should be understood as being defined in the frame obtained after the super-Weyl transformation, unless specified otherwise.)

The parameter Σ_E can be found by demanding that $K - 6\Sigma_E - 6\Sigma_E^\dagger$ have no lowest, Θ and Θ^2 components since this combination appears in the exponent of the first term in Eq. (3.2). To see, for example, why $(K - 6\Sigma_E - 6\Sigma_E^\dagger)|$ must vanish, note that Eq. (3.1) involves the lowest component of $e^{-K/3}$. If the lowest component of this term is scaled to 1, the factor $e^{-K/3}$ is absent and gravity is canonically normalized. The other two conditions lead to a

canonical kinetic term for the gravitino, and to canonical Kähler kinetic terms for the matter multiplets.

The conditions on Σ_E are, therefore,

$$|K| = 6|\Sigma_E| + 6|\Sigma_E^\dagger|, \quad |(\mathcal{D}_\alpha K)| = 6|(\mathcal{D}_\alpha \Sigma_E)|, \quad |(\mathcal{D}^2 K)| = 6|(\mathcal{D}^2 \Sigma_E)|, \quad (3.3)$$

or the vanishing of the lowest, Θ and Θ^2 components of $K - 6\Sigma_E - 6\Sigma_E^\dagger$. The Einstein frame conditions (3.3) almost completely determine the parameter Σ_E ,

$$\Sigma_E = A_\Sigma + \sqrt{2}\Theta\chi_\Sigma + \Theta^2 F_\Sigma, \quad (3.4)$$

with

$$A_\Sigma = \frac{1}{12}K + i\phi, \quad \chi_\Sigma = \frac{1}{6}K_i\chi^i, \quad F_\Sigma = \frac{1}{6}K_i F^i - \frac{1}{12}K_{ij}\chi^i\chi^j, \quad (3.5)$$

where the subscript i on K denotes the derivative with respect to A^i ($K_i \equiv \partial K / \partial A^i$), and F^i is the highest component of Φ^i . Note that the conditions (3.3) do not completely fix Σ_E ; the imaginary part ϕ of its lowest component is left undetermined. Also note that Γ does not contribute to the conditions (3.3): we assume that a field redefinition is performed so that the matter fields are in the Wess-Zumino gauge, where Γ has no lowest, Θ and Θ^2 components.

With the above Σ_E , it is not hard to find the component Lagrangian. If we substitute Eq. (3.4) into (3.2), all terms leading to non-Einstein gravity vanish. The rest of the component Lagrangian can be readily evaluated and gives the well-known Einstein supergravity Lagrangian with canonical kinetic terms. The complete expression for the Lagrangian is given in [1, 2]. We have seen that the component Lagrangian can be directly obtained from superspace, without any extra Weyl rescalings of the component fields.

Let us now show that we can safely set the field ϕ to zero. This field appears in the kinetic terms of the matter and gauge fermions,

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{kin}} = & -iK_{ij^*}\bar{\chi}^{j^*}\bar{\sigma}^a\left[\partial_a + \frac{i}{6}b_a - \frac{1}{6}\left(K_k\partial_a A^k - K_{j^*}\partial_a A^{j^*} - 12i\partial_a\phi\right)\right]\chi^i \\ & -i\bar{\lambda}\bar{\sigma}^a\left(\partial_a - \frac{i}{2}b_a\right)\lambda. \end{aligned} \quad (3.6)$$

(We omit terms with spin, sigma-model, and gauge connections because they are not relevant for our discussion.) The field ϕ also appears in the solution to the equations of motion for the auxiliary field b_a ,

$$b_a = \frac{i}{2}\left(K_j\partial_a A^j - K_{j^*}\partial_a A^{j^*} - 12i\partial_a\phi\right) + \frac{1}{4}K_{ij^*}\chi^i\sigma_a\bar{\chi}^{j^*} + \dots. \quad (3.7)$$

(For the complete expression for b_a , see Appendix B.) In addition, it appears in the superpotential terms in the Einstein frame,

$$e^{-1}\mathcal{L}_{\text{Yukawa}} = -\frac{1}{2}\exp\left(\frac{1}{2}K + 6i\phi\right)P_{ij}\chi^i\chi^j + \text{h.c.} + \dots. \quad (3.8)$$

(The complete expression for the Yukawa terms can be found in [1, 2].)

Upon inspection of Eqs. (3.6), (3.7) and (3.8), one can check that the classical component Lagrangian can be made independent of ϕ by redefining $\chi \rightarrow e^{-3i\phi}\chi$ and $\lambda \rightarrow e^{3i\phi}\lambda$. In fact, the ϕ dependence also cancels at the quantum level. The field redefinitions used to go to the Einstein frame are $\chi \rightarrow e^{-\text{Re}\Sigma_E + 3i\phi}\chi$ and $\lambda \rightarrow e^{-3\text{Re}\Sigma_E - 3i\phi}\lambda$. The Jacobian from these transformations exactly cancels the Jacobian from the redefinitions used to eliminate ϕ from the component Lagrangian. Therefore the field ϕ is unphysical, and we can safely set it to zero.

3.2 Supersymmetry Transformations in the Einstein Frame

In this subsection, we discuss the invariance of the classical supergravity Lagrangian in the Einstein frame. Then, in the next subsection, we consider quantum effects, and in particular, anomalies.

It is simple to see that the Einstein-frame Lagrangian is not invariant under the supersymmetry transformations (2.2). Under a supersymmetry transformation, the Kähler potential transforms as $\delta_{\text{SUSY}}K = -\xi^A \mathcal{D}_A K$, in which case Σ_E transforms to Σ'_E ,

$$\Sigma'_E \equiv \Sigma_E |_{K \rightarrow K + \delta_{\text{SUSY}}K} = \Sigma_E - \xi^A \mathcal{D}_A \Sigma_E - \Sigma_\xi, \quad (3.9)$$

where

$$\Sigma_\xi = -\frac{1}{6\sqrt{2}} \left(\xi \chi^i K_i - \bar{\xi} \bar{\chi}^{i*} K_{i*} \right) + \mathcal{O}(\Theta) + \mathcal{O}(\Theta^2) \equiv i\phi_\xi + \mathcal{O}(\Theta) + \mathcal{O}(\Theta^2). \quad (3.10)$$

The second term on the RHS of Eq. (3.9) is the supersymmetry transformation of a normal chiral superfield. The Σ_ξ term is an additional super-Weyl transformation under which the action is not invariant.

To see this explicitly, consider the supersymmetry transformation of the Einstein-frame Lagrangian,

$$\begin{aligned} \mathcal{L}_E(X + \delta_{\text{SUSY}}X) &= \int d^2\Theta \, 2\mathcal{E} \left[\frac{3}{8} (\bar{\mathcal{D}}^2 - 8R) \exp \left\{ -\frac{1}{3} \left(K - 6(\Sigma_E - \Sigma_\xi) - 6(\Sigma_E^\dagger - \Sigma_\xi^\dagger) + \Gamma \right) \right\} \right. \\ &\quad \left. + \frac{1}{4} H_{ab} W^{(a)} W^{(b)} + \exp \{ 6(\Sigma_E - \Sigma_\xi) \} P \right] + \text{h.c.} \end{aligned} \quad (3.11)$$

Since Σ_ξ is nonvanishing, the Lagrangian is not invariant under the supersymmetry transformation (2.2).

This lack of invariance stems from the fact that Σ_ξ takes the Lagrangian out of the Einstein frame. Invariance can be restored by returning to the Einstein frame through a compensating super-Weyl transformation. This is similar to gauge invariance in globally supersymmetric gauge theories. There, a supersymmetry transformation in the Wess-Zumino gauge must be supplemented by a superfield gauge transformation to restore the Wess-Zumino gauge

condition. It is instructive to consider this case in some detail because of the close analogy to supergravity [8]. To that end, we review the supersymmetry transformations of globally supersymmetric gauge theories in Appendix A.

For the case at hand, the compensating super-Weyl transformation has parameter Σ_ξ . In the Einstein frame, therefore, we *define* a supersymmetry transformation to include a frame-restoring super-Weyl transformation with parameter Σ_ξ :

$$\delta_\xi \equiv \delta_{\text{SUSY}} + \delta_{\text{SW}} . \quad (3.12)$$

Under such a transformation, chiral and vector superfields transform as follows,

$$\delta_\xi \Phi = -\xi^A \mathcal{D}_A \Phi + \delta_{\text{SW}} \Phi , \quad \delta_\xi V = -\xi^A \mathcal{D}_A V + \delta_{\text{SW}} V , \quad (3.13)$$

and analogously for the vielbein. In these expressions, the first terms on the RHS are the original supersymmetry transformations; the second are the compensating super-Weyl transformations with parameter Σ_ξ . These transformations eliminate the Σ_ξ in Eq. (3.11) and restore the classical invariance of the action,

$$\mathcal{L}_E(X + \delta_\xi X) = \mathcal{L}_E(X) . \quad (3.14)$$

The transformation properties of the individual component fields can be derived by expanding Eq. (3.13). We have checked that they agree with the transformations given in [1, 2] after eliminating the auxiliary fields.

3.3 Quantum Consistency in the Einstein Frame

We are now ready to discuss anomalies in the supersymmetry transformations (3.12). As we have seen, these transformations include frame-restoring super-Weyl field rescalings that induce chiral rotations on the matter fermions,

$$\delta_\xi \chi = \cdots + 3i\phi_\xi \chi , \quad \delta_\xi \lambda = \cdots - 3i\phi_\xi \lambda , \quad (3.15)$$

where $i\phi_\xi = \Sigma_\xi$. At the quantum level, these transformations are anomalous, so they should give rise to an anomalous variation of the 1PI Lagrangian,

$$\delta_\xi \mathcal{L}_{\text{1PI}} = e \left[\frac{1}{16\pi^2} (3T_R - 3T_G) \phi_\xi F_{mn}^{(a)} \tilde{F}^{mn(a)} + \cdots \right] , \quad (3.16)$$

If nothing were to cancel this variation, local supersymmetry in the Einstein frame would be anomalous. In what follows, we will show that the full 1PI effective action is, in fact, invariant. The variation (3.16) is cancelled by the variation of the Jacobian (2.14) that arises in passing to the Einstein frame.

In the Einstein frame, the complete 1PI effective Lagrangian is of the following form,

$$\mathcal{L}_{\text{1PI}} = \mathcal{L}_E + \Delta \mathcal{L} + \mathcal{L}_J . \quad (3.17)$$

The first term is the classical part of the Einstein-frame Lagrangian, the second is the non-local term induced by anomalies, and the third is the Jacobian (2.14). The first term is invariant under the local supersymmetry transformation (3.12), as discussed in the previous subsection. The second and third terms are not. Under the supersymmetry transformation (3.12), the nonvanishing variation of $\Delta\mathcal{L}$ expresses the anomaly associated with the frame-restoring super-Weyl transformation.^{#5} If this variation were the only change of the Lagrangian, supersymmetry would be explicitly broken by anomalies at the quantum level.

Fortunately, however, it is not. There is also \mathcal{L}_J , the Jacobian that arises in the Einstein frame. This term is *not* invariant under (3.12). From Eq. (3.9), we have

$$\delta_\xi \Sigma_E = -\xi^A \mathcal{D}_A \Sigma_E - \Sigma_\xi. \quad (3.18)$$

The first term on the RHS is the supersymmetry transformation of a normal chiral superfield. The second is a super-Weyl transformation of Σ . This gives

$$\begin{aligned} \delta_\xi \mathcal{L}_J &= -\frac{1}{16\pi^2} (3T_R - 3T_G) \int d^2\Theta \, 2\mathcal{E} \, \Sigma_\xi W^{(a)} W^{(a)} + \text{h.c.} \\ &= e \left[-\frac{1}{16\pi^2} (3T_R - 3T_G) \phi_\xi F_{mn}^{(a)} \tilde{F}^{mn(a)} + \dots \right]. \end{aligned} \quad (3.19)$$

Equation (3.19) exactly cancels the variation (3.16) and restores supersymmetry invariance in the Einstein frame.

Thus we have seen that the Einstein-frame 1PI effective Lagrangian is invariant under local supersymmetry transformations – *provided* the Jacobian (2.14) is added to the bare Lagrangian. Otherwise, local supersymmetry is explicitly broken at the quantum level because of the anomaly associated with the frame-restoring super-Weyl transformations. The Jacobian (2.14) is essential for the consistency of the quantum theory. The component expression for the Jacobian is given in Appendix B.

4 Physical Implications of the Jacobian: An Example

In this section, we show that the anomalous Jacobian \mathcal{L}_J has physical consequences. We illustrate this with an example of a scattering amplitude which requires the Jacobian to give a frame-independent result.

In what follows we consider a model with a no-scale Kähler potential of the form

$$K = -3 \log \left(1 - \frac{1}{3} \Phi^\dagger \Phi \right). \quad (4.1)$$

^{#5}If we make the argument before integrating out the light fields, $\Delta\mathcal{L}$ does not exist. In this case the anomaly arises as a change of the functional measure of the path integral.

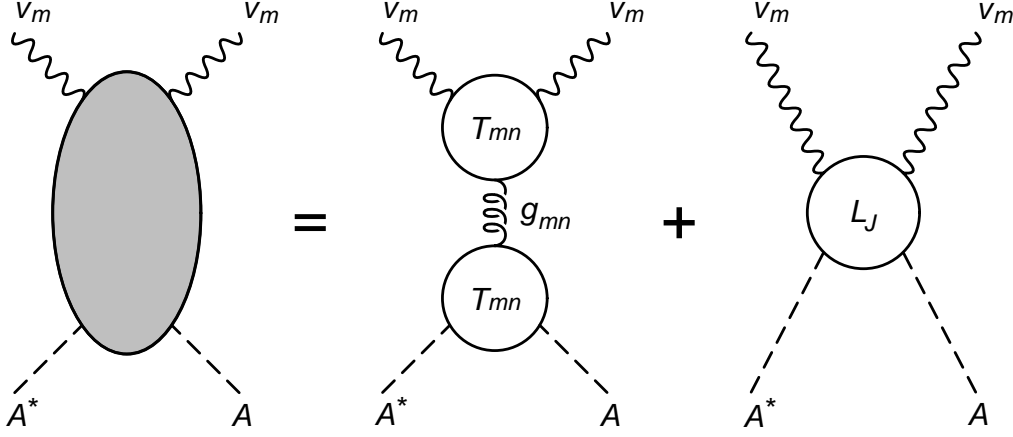


Figure 1: Schematic picture of the amplitude calculation.

This Kähler potential (4.1) is chosen for simplicity; upon substitution into the superspace Lagrangian (2.1), it gives rise to canonically normalized scalars with a conformal coupling to gravity,

$$\mathcal{L}_{\text{SUGRA}} = \sqrt{-g} \left[-\frac{1}{2} \left(1 - \frac{1}{3} A^* A \right) \mathcal{R} - g^{mn} \partial_m A^* \partial_n A + \dots \right], \quad (4.2)$$

where we used the metric g_{mn} instead of a vielbein. In Eq. (4.2), the subscript SUGRA indicates that this is the supergravity-frame Lagrangian. As discussed in the previous sections, we can use a super-Weyl transformation to pass to the Einstein frame.

In this model, we study the scattering process $A^* A \rightarrow v_m v_m$ via a graviton exchange, where v_m is a gauge boson and A is a massless neutral scalar field. For simplicity, we assume that the gauge theory is pure supersymmetric Yang Mills.^{#6}

The amplitude of this process can be written as follows (see Fig. 1)

$$\begin{aligned} i\mathcal{M}(A^* A \rightarrow v_m v_m) &= \frac{i}{2} \langle v_m(\epsilon_1, p'_1) v_m(\epsilon_2, p'_2) | T^{kl} | 0 \rangle \times i\Delta_{kl, mn} \\ &\quad \times \frac{i}{2} \langle 0 | T^{mn} | A^*(p_1) A(p_2) \rangle \\ &\quad + \langle v_m(\epsilon_1, p'_1) v_m(\epsilon_2, p'_2) | i\mathcal{L}_J | A^*(p_1) A(p_2) \rangle, \end{aligned} \quad (4.3)$$

where the p_i denote the momenta of the scalars, while ϵ_i and p'_i denote the polarizations and momenta of the gauge bosons. Here, T_{mn} is the energy momentum tensor,

$$T_{mn} = \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}}{\delta g^{mn}}, \quad (4.4)$$

^{#6}If the matter fields had gauge quantum numbers, the following discussion would still hold, provided we replace $3T_G$ by $3T_G - T_R$.

while $\Delta_{kl,mn}$ is the graviton propagator, given by

$$\Delta_{mn,kl}(q) = -\frac{2}{q^2}(\eta_{km}\eta_{ln} + \eta_{kn}\eta_{lm} - \eta_{kl}\eta_{mn}) , \quad (4.5)$$

where η_{mn} is the flat-space metric. We omit the part that depends on the gauge parameter because it does not contribute to the amplitude of interest.

For the scalars, the matrix element of the energy-momentum tensor is

$$\langle 0 | T^{mn} | A^*(p_1) A(p_2) \rangle = p_1^m p_2^n + p_1^n p_2^m - (p_1 p_2) \eta^{mn} + \frac{\zeta}{3} (q^2 \eta^{mn} - q^m q^n) , \quad (4.6)$$

where $q \equiv p_1 + p_2$, and ζ is a coefficient proportional to the coupling of the scalars to the scalar curvature \mathcal{R} ; $\zeta = 1$ in the supergravity frame and $\zeta = 0$ in the Einstein frame. Note that $\zeta = 1$ corresponds to conformally coupled scalars; this is easy to see upon computing the trace of Eq. (4.6) with $q^2 = 2p_1 p_2$.

Substituting Eqs. (4.5) and (4.6) into Eq. (4.3), we obtain the amplitude

$$\begin{aligned} \mathcal{M} &= \frac{2}{q^2} p_{1k} p_{2l} \langle v_m v_m | T^{kl} | 0 \rangle + \frac{\zeta}{6} \langle v_m v_m | T_k^k | 0 \rangle + \langle v_m v_m | \mathcal{L}_J | A^* A \rangle \\ &\equiv \mathcal{M}_0 + \frac{\zeta}{6} \langle v_m v_m | T_k^k | 0 \rangle + \langle v_m v_m | \mathcal{L}_J | A^* A \rangle , \end{aligned} \quad (4.7)$$

where \mathcal{M}_0 is frame independent. At the classical level, with \mathcal{L}_J absent, the amplitudes in both frames are identical because the gauge boson energy momentum tensor is traceless.

At the quantum level, the frame independence is more subtle. First, the gauge boson energy momentum tensor is no longer traceless. Second, there is a Jacobian in the Einstein frame, but not the supergravity frame. To see what happens, let us consider the supergravity frame. We take $\zeta = 1$ and use the trace anomaly relation

$$T_k^k = \frac{3}{32\pi^2} T_G F_{mn}^{(a)} F^{mn(a)} , \quad (4.8)$$

to find

$$\mathcal{M}_{\text{SUGRA}} = \mathcal{M}_0 + \frac{3}{192\pi^2} T_G \langle v_m v_m | F_{mn}^{(a)} F^{mn(a)} | 0 \rangle . \quad (4.9)$$

In the Einstein frame, where $\zeta = 0$, there is no contribution from the T_k^k term. However, in this frame there is the super-Weyl Jacobian,

$$\begin{aligned} \mathcal{L}_J &= \frac{1}{16\pi^2} (-3T_G) \int d^2\Theta \, 2\mathcal{E} \, \Sigma_E W^{(a)} W^{(a)} + \text{h.c.} \\ &= \frac{3}{192\pi^2} T_G A^* A F_{mn}^{(a)} F^{mn(a)} + \dots , \end{aligned} \quad (4.10)$$

where we have used $|\Sigma_E| = \frac{1}{12}A^*A + \dots$. With this Jacobian, the Einstein-frame matrix element is

$$\mathcal{M}_E = \mathcal{M}_0 + \frac{3}{192\pi^2} T_G \langle v_m v_m | F_{mn}^{(a)} F^{mn(a)} | 0 \rangle , \quad (4.11)$$

which is in complete agreement with the amplitude in the supergravity frame. The results in the two frames are identical because of the super-Weyl Jacobian.

5 Summary

In this paper, we studied the quantum consistency of the supergravity Lagrangian. We used a superspace approach in which the supergravity Lagrangian does not automatically give canonically normalized Einstein gravity. In the literature, Einstein gravity is recovered after a redefinition of the component fields. In this paper, we showed that the field redefinition is, in fact, a super-Weyl transformation, and we demonstrated a systematic way to do the field redefinition in superspace. This approach provides us with a clear understanding of supersymmetry transformations in the Einstein frame.

Supersymmetry transformations in the Einstein frame must preserve the Einstein frame condition, so they differ from the original supersymmetry transformations defined in the supergravity frame. We showed that the Einstein-frame supersymmetry transformations are ordinary supersymmetry transformations, δ_{SUSY} , combined with compensating super-Weyl transformations, δ_{SW} , which are necessary to maintain the Einstein frame condition.

The compensating super-Weyl transformations are, at the quantum level, anomalous. Because of this fact, one must be careful when studying the supersymmetry invariance of the quantum effective action. In this paper we emphasized that the super-Weyl transformation used to pass to the Einstein frame is anomalous. It gives rise to an anomalous Jacobian that must be included in the bare Einstein-frame Lagrangian. The 1PI Einstein-frame Lagrangian is supersymmetric because the variation of this Jacobian precisely cancels the anomaly arising from the frame-restoring super-Weyl transformation. If the Jacobian were omitted, the 1PI Lagrangian would not be invariant under Einstein-frame supersymmetry transformations. Consistency demands that the Jacobian be included in the bare Einstein-frame Lagrangian.

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A Globally Supersymmetric Gauge Theories

In this Appendix, we show how the supersymmetry transformations are defined in globally supersymmetric gauge theories. In particular, we demonstrate how the Wess-Zumino gauge condition is maintained after supersymmetry transformations. We will see that there is a close analogy between supersymmetry transformations in globally supersymmetric gauge theories and supergravity transformations in the Einstein frame.

Consider a globally supersymmetric gauge theory, with the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \int d^4\theta \Phi^\dagger e^V \Phi, \quad (\text{A.1})$$

where $\mathcal{L}_{\text{gauge}}$ is the kinetic term for the gauge multiplet. This Lagrangian is invariant under the superspace supersymmetry transformations

$$\delta_{\text{SUSY}} V = \xi^A \partial_A V, \quad \delta_{\text{SUSY}} \Phi = \xi^A \partial_A \Phi, \quad (\text{A.2})$$

where $\xi^a = -i\xi\sigma^a\bar{\theta} + i\theta\sigma^a\bar{\xi}$ and ξ^α is the supersymmetry transformation parameter. The component transformations can be determined by expanding (A.2) in powers of θ .

The Lagrangian (A.1) contains the lower components of V , which are gauge degrees of freedom. Usually, it is convenient to use a Lagrangian in which these lower components are eliminated by a gauge transformation. This “frame” is often called “the Wess-Zumino gauge;” it is obtained by the following field redefinitions:

$$V_{\text{WZ}} = V - \Lambda - \Lambda^\dagger, \quad \Phi_{\text{WZ}} = e^\Lambda \Phi, \quad (\text{A.3})$$

where Λ is a chiral superfield. The gauge parameter Λ is chosen so that all the unphysical fields are eliminated from the Lagrangian. The conditions are

$$V| = \Lambda| + \Lambda^\dagger|, \quad (D_\alpha V)| = (D_\alpha \Lambda)|, \quad (D^2 V)| = (D^2 \Lambda)|, \quad (\text{A.4})$$

so Λ in the chiral basis is simply

$$\Lambda = \frac{1}{2}C + i\phi + i\theta\chi + \frac{i}{2}\theta^2(M + iN), \quad (\text{A.5})$$

where C , χ , $M + iN$ are the lowest, θ and θ^2 components of the vector superfield V , respectively. Note that the field ϕ is not determined by the Wess-Zumino gauge conditions; it is the parameter of an ordinary gauge transformation. In terms of the new variables, the Lagrangian becomes

$$\begin{aligned} \mathcal{L}_{\text{WZ}} &= \mathcal{L}_{\text{gauge}} + \int d^4\theta \Phi_{\text{WZ}}^\dagger e^{V-\Lambda-\Lambda^\dagger} \Phi_{\text{WZ}} \\ &= \mathcal{L}_{\text{gauge}} + \int d^4\theta \Phi_{\text{WZ}}^\dagger e^{V_{\text{WZ}}} \Phi_{\text{WZ}}. \end{aligned} \quad (\text{A.6})$$

The Wess-Zumino gauge Lagrangian \mathcal{L}_{WZ} contains only the physical fields.

The Wess-Zumino gauge conditions, however, are not preserved by the supersymmetry transformations (A.2). They must be supplemented by compensating superfield gauge transformations,

$$\delta_\xi V_{\text{WZ}} = \xi^A \partial_A V_{\text{WZ}} + \Lambda_\xi, \quad \delta_\xi \Phi_{\text{WZ}} = \xi^A \partial_A \Phi_{\text{WZ}} + \Lambda_\xi \Phi_{\text{WZ}}, \quad (\text{A.7})$$

where, in the chiral basis,

$$\Lambda_\xi = \theta \sigma^m \bar{\xi} (-2v_m + 2\partial_m \phi) + \theta^2 \bar{\xi} \bar{\lambda}. \quad (\text{A.8})$$

Here v_m and λ are the gauge boson and gaugino fields, respectively.

The transformations (A.7) are combinations of the original supersymmetry transformations (A.2) and the frame-restoring gauge transformations. They leave invariant the Wess-Zumino-gauge Lagrangian. Furthermore, if we expand the LHS of (A.7) in powers of θ , we obtain the Wess-Zumino-gauge supersymmetry transformations given, for example, in [2]. Note that in Wess-Zumino gauge, a theory with a gauge anomaly would also have a supersymmetry anomaly because the transformations (A.2) contain ordinary gauge transformations.

Finally, we comment on the difference between (A.8) and (3.10), that is, on the different way we treat the imaginary part of the lowest component of the compensating transformation parameters. In each case, the term is not determined by the Wess-Zumino gauge/Einstein frame conditions. In globally supersymmetric gauge theory, we choose not to fix ϕ_ξ in Λ_ξ ; it is the degree of freedom associated with an ordinary gauge transformation. By contrast, in supergravity, we completely fix it and demand that the imaginary part of Σ_{E} vanish. The first term in (3.10) ensures that the imaginary part of Σ_{E} does not reappear in the Lagrangian after a supersymmetry transformation.

B Component Expression for the Jacobian

In this Appendix, we present the complete component expression for the Jacobian that arises from the super-Weyl transformation required to pass to the Einstein frame.

As we have seen in this paper, the bare Lagrangian in Einstein-frame supergravity is given by

$$\hat{\mathcal{L}}_{\text{bare}} = \mathcal{L}_{\text{E}} + \mathcal{L}_{\text{J}} \quad (\text{B.1})$$

where \mathcal{L}_{E} is the classical supergravity Lagrangian whose component expression is given, for example, in [1, 2]. \mathcal{L}_{J} is the Jacobian. At one-loop level, \mathcal{L}_{J} is given by

$$\mathcal{L}_{\text{J}} = \frac{1}{16\pi^2} (3T_R - 3T_G) \int d^2\Theta \, 2\mathcal{E} \, \Sigma_{\text{E}} W^{(a)} W^{(a)} + \text{h.c.}, \quad (\text{B.2})$$

where the chiral superfield Σ_{E} is given in Eqs. (3.4) and (3.5) with $\phi = 0$,

$$\Sigma_{\text{E}} = A_{\Sigma} + \sqrt{2}\Theta\chi_{\Sigma} + \Theta^2 F_{\Sigma}, \quad (\text{B.3})$$

with

$$A_\Sigma = \frac{1}{12}K, \quad \chi_\Sigma = \frac{1}{6}K_i\chi^i, \quad F_\Sigma = \frac{1}{6}K_iF^i - \frac{1}{12}K_{ij}\chi^i\chi^j. \quad (\text{B.4})$$

Expanding Eq. (B.2), we obtain

$$\begin{aligned} e^{-1}\mathcal{L}_J = & \frac{1}{16\pi^2} (3T_R - 3T_G) \left[-A_\Sigma F_{mn}^{(a)} F^{mn(a)} \right. \\ & - 2iA_\Sigma \lambda^{(a)} \sigma^m \left(\mathcal{D}_m \bar{\lambda}^{(a)} - f^{abc} v_m^{(b)} \bar{\lambda}^{(c)} + \frac{i}{2} b_m \bar{\lambda}^{(a)} \right) \\ & - 2iA_\Sigma \bar{\lambda}^{(a)} \bar{\sigma}^m \left(\mathcal{D}_m \lambda^{(a)} - f^{abc} v_m^{(b)} \lambda^{(c)} - \frac{i}{2} b_m \lambda^{(a)} \right) \\ & + 2A_\Sigma D_{\text{aux}}^{(a)} D_{\text{aux}}^{(a)} \\ & + iA_\Sigma \left(\psi_m \sigma^{kl} \sigma^m \bar{\lambda}^{(a)} + \bar{\psi}_m \bar{\sigma}^{kl} \bar{\sigma}^m \lambda^{(a)} \right) \left(F_{kl}^{(a)} + \hat{F}_{kl}^{(a)} \right) \\ & - \sqrt{2} \left(\chi_\Sigma \sigma^{mn} \lambda^{(a)} + \bar{\chi}_\Sigma \bar{\sigma}^{mn} \bar{\lambda}^{(a)} \right) F_{mn}^{(a)} \\ & + \sqrt{2}i \left(\chi_\Sigma \sigma^{mn} \lambda^{(a)} \psi_m \sigma_n \bar{\lambda}^{(a)} + \frac{1}{4} \bar{\psi}_m \bar{\sigma}^m \chi_\Sigma \lambda^{(a)} \lambda^{(a)} \right) \\ & + \sqrt{2}i \left(\bar{\chi}_\Sigma \bar{\sigma}^{mn} \bar{\lambda}^{(a)} \bar{\psi}_m \bar{\sigma}_n \lambda^{(a)} + \frac{1}{4} \psi_m \sigma^m \bar{\chi}_\Sigma \bar{\lambda}^{(a)} \bar{\lambda}^{(a)} \right) \\ & + \sqrt{2}i \left(\chi_\Sigma \lambda^{(a)} - \bar{\chi}_\Sigma \bar{\lambda}^{(a)} \right) D_{\text{aux}}^{(a)} \\ & \left. - F_\Sigma \lambda^{(a)} \lambda^{(a)} - F_\Sigma^* \bar{\lambda}^{(a)} \bar{\lambda}^{(a)} \right] \end{aligned} \quad (\text{B.5})$$

where $\hat{F}_{mn}^{(a)}$ is the supercovariant field strength,

$$\hat{F}_{mn}^{(a)} = F_{mn}^{(a)} - \frac{i}{2} \left(\psi_m \sigma_n \bar{\lambda}^{(a)} + \bar{\psi}_m \bar{\sigma}_n \lambda^{(a)} - \psi_n \sigma_m \bar{\lambda}^{(a)} - \bar{\psi}_n \bar{\sigma}_m \lambda^{(a)} \right). \quad (\text{B.6})$$

For a detailed explanation of the notation, see [2].

The full component expression is given by substituting the solutions to the Einstein-frame auxiliary field equations of motion,

$$\begin{aligned} F^i &= (K^{-1})^{ij*} \left(-e^{K/2} D_{j*} P^* + \frac{1}{2} K_{j*kl} \chi^k \chi^l + \frac{1}{4} \partial_{j*} h_{(ab)} \bar{\lambda}^{(a)} \bar{\lambda}^{(b)} \right) \\ D_{\text{aux}}^{(a)} &= -h^{\text{R}(ab)-1} \left[D^{(b)} + \frac{i}{2\sqrt{2}} \left(\partial_i h_{(bc)} \chi^i \lambda^{(c)} - \partial_{i*} h_{(bc)} \bar{\chi}^i \bar{\lambda}^{(c)} \right) \right] \\ b_m &= \frac{i}{2} \left(K_i \tilde{\mathcal{D}}_m A^i - K_{i*} \tilde{\mathcal{D}}_m A^{*i} \right) + \frac{1}{4} K_{ij*} \chi^i \sigma_m \bar{\chi}^j \\ &\quad - \frac{3}{4} h_{(ab)}^{\text{R}} \lambda^{(a)} \sigma_m \bar{\lambda}^{(b)} + i \left[\frac{1}{2} \left(K_i X^{i(a)} - K_{i*} X^{*i(a)} \right) + i D^{(a)} \right] v_m^{(a)}, \end{aligned} \quad (\text{B.7})$$

where $D_i P \equiv \tilde{D}_i P + K_i P$, $X^{(a)}$ is the Killing vector, $D^{(a)}$ is the Killing potential associated with $X^{(a)}$, and $\tilde{\mathcal{D}}_m A^i \equiv \mathcal{D}_m A^i - v_m^{(a)} X_{(a)}^i$ [2].

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